

HW2 , Math 531, Spring 2014

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QUESTION 1. (i) Let R be a finite ring (with more than one element) and with no nonzero zero-divisors. Prove that R must have an identity.

(ii) Let R be a finite ring (with more than one element) and with no nonzero zero-divisors. Prove that R is a division ring.

(iii) Let R be a ring with one (not necessarily commutative) and with no nonzero zero-divisors. Show that $char(R) = p$ (prime) or 0

(iv) Let $R = Z_8(+)Z_8$. For $(a, b), (c, d) \in R$, define $(a, b) + (c, d) = (a + c, b + d)$ and $(a, b).(c, d) = (ac, bc + ad)$. Then we know that R is a commutative ring with 1. Show the following

a. Find all nonzero zero-divisors of R.

b. Find all nilpotent elements of R.

(v) Let R be a commutative ring with 1, and $w \in Nil(R)$. Prove that $1 + w \in U(R)$. Then prove that $u + w \in U(R)$ for every $u \in U(R)$. [Hint: note that $u + w = (1 + wu^{-1})u$ and surely $wu^{-1} \in Nil(R)$. You may need to use some basic elementary algebra facts, for example: $x^3 + 1 = (x + 1)(x^2 - x + 1)$]

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